MATHEMATICS PAPER IIA - MARCH 2009.

ALGEBRA AND PROBABILITY

TIME : 3hrs **Max. Marks.75**

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Noe : Attempt all questions. Each question carries 2 marks.

1. If α and β are the roots of the equation then find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

2. Find the algebraic equation whose roots are 3 times the roots of $x^3+2x^2-4x+1=0$

3. Find the adjoint and the inverse of the matrix $\begin{bmatrix}\cos\alpha & -\sin \\ \sin\alpha & \cos\alpha\end{bmatrix}$

4. If $A=\begin{bmatrix}0 & 1 & 4 \\ -1 & 0 & 7 \\ -X & -7 & 0\end{bmatrix}$ is a skew symmetric matrix, then find x.

5. If ${}^{n}P_{r}=1320$ find r.

6. Find the number of ways of arranging the letters of the word INDEPENDENCE

7. Find the coefficient of x^7 in the expansion of $\left(\frac{3x^2}{7}+\frac{4}{5x^3}\right)^{11}$

8. Find the sum of the infinite series $\frac{1}{2}-\frac{1}{2\cdot 2^2}+\frac{1}{3\cdot 2^3}-\frac{1}{4\cdot 2^4}+\ldots$
9. Find the probability that a non leap year contains 53 Sundays.
10. A Poisson variable satisfies P(X = 1) = 3P(X = 2) , then find the variance of X

SECTION B

SHORT ANSWER TYPE QUESTIONS. **5X4 =20**

Note : Answer any FIVE questions. Each question carries 4 marks.

11. Find the range of the following expression if $\frac{2x^2 - 6x + 5}{x^2 - 3x + 5}$

12. If $3A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$ then show that $A^{-1} = A^{1}$

13. Find the numberof ways of formaing a committee of 5 members out 6 Indians and 5 Americans so that always the Indians will be majority in the committee

14. If the letters of the word MASTER are permuted an all possible ways and the words thus formed are arranged in the dictionary order. Then find the rank of the word MASTER

15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.

16. Find the sum of the infinite series $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....$

17. If A and B are independent events with $P(A) = 0.2, P(B) = 0.5$, then find i) $P(A \cap B), P(A \cup B)$, $P(A|B), P(B|A)$

SECTION C

LONG ANSWER TYPE QUESTIONS. **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. Given that the roots of $x^3 + px^2 + qx + r = 0$ *are in* A.P., show that $2p^3 - 9qp + 27r = 0$

19. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc=ca)$

20. Solve the following simultaneous linear equation by using

Gauss - Jordean method x + y + z = 9, 2x + 5y + 7z = 52, 2x + y – z = 0

21. State and prove Binomial theorem.

22. If $x = \frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+...........$ then prove that $9x^2 + 24x = 11$

23. (i) State Baye’s theorem.

(ii) There are 3 black and 4 white balls in bag B_1. 4 black and 3 white in the bag B_2. A die is rolled and the first bag is selected if it is 1 or 3, and the second bag for the rest. F the ball drawn is found blace, then find the probability that it is drawn from bag B_1.

24. A random variable X has the following probability distribution

X = x	0	1	2	3	4	5	6	7
P (X = x)	0	k	2k	3k	k^2	$2k^3$	$7k^2 + k$	

Find (i) k (ii) then mean and (iii) P (0 < X < 5)