MATHEMATICS PAPER IA.- MARCH 2010.

ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY

TIME : 3hrs **Max. Marks.75**

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Noe : Attempt all questions. Each question carries 2 marks.

1. If $f : Q \to Q$ is defined by $f(x) = 5x + 4$ for all $x \in Q$, find f^{-1}

2. Find the domain of the function $f(x) = \log(x^2 - 4x + 3)$

3. Let **a = 2i + 4j – 5k, b = i + j + k** and **c = j + 2k**. Find unit vector in the opposite direction of **a + b + c**.

4. Find the vector equation to the line passing through the points i+j+k, i-j+k

5. Find the value of $\cos^2 45^0 - \sin^2 15^0$

6. Find the value of $\tan 10^0 + \tan 35^0 + \tan 10^0 . \tan 35^0$

7. If $\cosh x = \frac{5}{2}$, find the values of i) cosh (2x) and ii) sinh (2x)

8. In a ΔABC, find $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$

9. Write the polar form of -1+i

10. Find the angle between the vectors i+2j+3k and 3i-j+2k

SECTION B

SHORT ANSWER TYPE QUESTIONS. **5X4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. If $\bar{a}, \bar{b}, \bar{c}$ are linearly independent vectors, then show that $\bar{a} - 2\bar{b} + 3\bar{c}, -2\bar{a} + 3\bar{b} - 4\bar{c}, -\bar{b} + 2\bar{c}$ are linearly dependent

12. Prove that the angle between any two diagonals of a cube is given by $\cos^{-1}\frac{1}{3}$.

13. If $\tan\theta = \frac{b}{a}$, then prove that $a\cos 2\theta + b\sin 2\theta = a$

14. Solve the $\sin x + \sqrt{3}\cos x = \sqrt{2}$

15. Prove that $Sin^{-1}\frac{4}{5} + 2Tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

16. In ΔABC, prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

17. Show that $8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$

SECTION C

LONG ANSWER TYPE QUESTIONS. **5 × 7 = 35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. If $f : A \to B,\ g : B \to C$ be bijections. Then prove that $(gof)^{-1} = f^{-1}og^{-1}$.

19. Prove by induction $a + (a + d) + (a + 2d) + ...to\, n\, terms = \frac{n}{2}[2a + (n-1)d]$

20. Find the cartesian equation of the plane passing through the points (2, 3, 1), (4, 5, 2), and (3, 6, 5).

21. In triangle ABC, prove that $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{\pi - A}{4}\cos\frac{\pi - B}{4}\cos\frac{\pi - C}{4}$

22. If P_1, P_2, P_3 are altitudes drawn from vertices A,B,C to the opposite sides of a triangle respectively, then show that

(i) $\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} = \frac{1}{r}$ (ii) $P_1P_2P_3 = \frac{(abc)^2}{8R^3}$

23. On a tower AB of height h, there is a flag – staff BC. At a point d meters away from the foot of the tower, AB and BC are making equal angles. Show that the height of the flag – staff is $h\left(\frac{d^2 + h^2}{d^2 - h^2}\right)$ meters.

24. If n is an integer then show that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$

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