MATHEMATICS PAPER IA- MAY 2009.

ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY

TIME: 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A, B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Note: Attempt all questions. Each question carries 2 marks.

1. Find the domain of the following real valued function $f(x)=\sqrt{16-X^2}$

2. If $f:R\to R, g:R\to R$ defined by $f(x)=3x-1, g(x)=x^2+1$ then find
   (i) $(fog)(x)$, (ii) $(gof)(x)$

3. Let A B C D E F be a regular hexagon with centre ‘O’. Show that
   AB + **AC** + **AD** + **AE** + **AF**=3 **AD**=6 **AO**

4. Find the Cartesian equation of the line joining the points 2**i + j +** 3**k** and **–** 4**i** + 3**j – k.**

5. Find unit vector perpendicular to the plane determined by 4 $\vec{a}=4\bar{i}+3\bar{j}-\bar{k}$ $and\ \overrightarrow{b}=2\bar{i}-6\bar{j}-3\bar{k}$

6. Find the maximum and minimum values of the function f(x) = 5sinx+12cosx+13 over R.

7. If $A+B=45^o$ then prove that $(1+\tan A)(1+\tan B)=2$.

8. If $\sinh x=\frac{3}{4}$, find $\cosh(2x)$ and $\sinh(2x)$.

9. In ΔABC, a=4, b=5, c=7 then find the value of cosB/2.

10. Find the square root of 3+4i.

SECTION B

SHORT ANSWER TYPE QUESTIONS **5X4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. Show that the points 7j+ 10k, – i+6j+6k, – 4i+9j+6k form a right-angled isosceles triangle.
12. Find the volume of the tetrahedron having the edges **i + j + k, i – j** and **i +** 2**j + k.**
13. Prove that $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}$
14. Solve $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$
15. prove that $Sin^{-1}\frac{3}{5}+Sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$
16. Show that in DABC, a = b cos c + c cos B.
17. Show that $2^6\sin^4\theta\cos^3\theta=\cos7\theta-\cos5\theta-3\cos3\theta+3\cos\theta$.

SECTION C

LONG ANSWER TYPE QUESTIONS. **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. If $f:A\to B,\ g:B\to C$ be bijections. Then prove that $(gof)^{-1}=f^{-1}og^{-1}$.

19. Using mathematical induction prove that $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+\ldots$ upto n terms
$=\frac{n(n+1)^2(n+2)}{12}$
20. For any vectors a,b,c prove that $\left(\bar{a}\times\bar{b}\right)\times\bar{c}=\left(\bar{a}.\bar{c}\right)\bar{b}-\left(\bar{b}.\bar{c}\right)\bar{a}$.
21. If A, B, C are angles in a triangle, then prove that cos A + cos B + cos C = 1 + 4 sin $\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.
22. In any triangle ABC, Show that $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$.
23. On a tower AB of height h, there is a flag – staff BC. At a point d meters away from the foot of the tower, AB and BC are making equal angles. Show that the height of the flag – staff if $h\left(\frac{d^2+h^2}{d^2-h^2}\right)$ meters.
24. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}$