Mathematics

1. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having α/β and β/α as its roots, is :
(a) $3x^2 + 19x + 3 = 0$ (b) $3x^2 - 19x + 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 16x + 1 = 0$

2. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is :
(a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$

3. If 1, $\log_3\sqrt{(3^{1-x} + 2)}$, $\log_3(4\cdot 3^x - 1)$ are in AP, then x equals :
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved, is :
(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3

5. The period of $\sin^2\theta$ is :
(a) π^2 (b) π
(c) 2π (d) $\pi/2$

6. l, m, n are the pth, qth and rth term of an GP and all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :
(a) 3 (b) 2
(c) 1 (d) zero

7. $\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}\,x}$ is :
(a) λ (b) -1
(c) zero (d) does not exist

8. A triangle with vertices (4, 0), (–1, –1), (3, 5) is :
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

9. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?
(a) 73 (b) 65
(c) 68 (d) 74

10. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to :
(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$
(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

11. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are :
(a) $\left(1, \frac{2}{3}\right)$ (b) (3, 1)
(c) (3, 3) (d) (1, 2)

12. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is :
(a) $x - y + z = 1$ (b) $x + y + z = 5$
(c) $x + 2y - z = 1$ (d) $2x - y + z = 5$

13. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is :
(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$

14. $\lim_{x \to \infty}\left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ is equal to :
(a) e^4 (b) e^2
(c) e^3 (d) e

15. The domain of $\sin^{-1}[\log_3(x/3)]$ is :
(a) [1, 9] (b) [–1, 9]
(c) [–9, 1] (d) [–9, –1]

16. The value of $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/16}\ldots\infty$ is :
(a) 1 (b) 2
(c) 3/2 (d) 4

17. Fifth term of a GP is 2, then the product of its 9 terms is :
(a) 256 (b) 512
(c) 1024 (d) none of these

18. $\int_0^{10\pi} |\sin x|\,dx$ is :
(a) 20 (b) 8
(c) 10 (d) 18

19. $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals :

(a) $\frac{1}{2}$ (b) 1

(c) ∞ (d) zero

20. $\int_0^2 [x^2]\, dx$ is :

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

21. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\, dx$ is :

(a) $\frac{\pi^2}{4}$ (b) π^2

(c) zero (d) $\frac{\pi}{2}$

22. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is :

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) none of these

23. The domain of definition of the function

$$f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)} \text{ is :}$$

(a) [1, 4] (b) [1, 0]

(c) [0, 5] (d) [5, 0]

24. If $\sin y = x \sin(a+y)$, then $\frac{dy}{dx}$ is :

(a) $\frac{\sin a}{\sin^2(a+y)}$ (b) $\frac{\sin^2(a+y)}{\sin a}$

(c) $\sin a \sin^2(a+y)$ (d) $\frac{\sin^2(a-y)}{\sin a}$

25. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is :

(a) $\frac{1+x}{1+\log x}$ (b) $\frac{1-\log x}{1+\log x}$

(c) not defined (d) $\frac{\log x}{(1+\log x)^2}$

26. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$:

(a) cut at right angle (b) touch each other

(c) cut at an angle $\frac{\pi}{3}$ (d) cut at an angle $\frac{\pi}{4}$

27. The function $f(x) = \cot^{-1} x + x$ increases in the interval :

(a) $(1, \infty)$ (b) $(-1, \infty)$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$

28. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0, 1] is :

(a) 1 (b) 2

(c) 3 (d) 1/3

29. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) zero (d) 1

30. $\int \frac{dx}{x(x^n+1)}$ is equal to :

(a) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$

(b) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right) + c$

(c) $\log\left(\frac{x^n}{x^n+1}\right) + c$

(d) none of these

31. The area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is given by :

(a) $\frac{9}{2}$ sq unit (b) $\frac{43}{6}$ sq unit

(c) $\frac{35}{6}$ sq unit (d) none of these

32. The differential equation of all non-vertical lines in a plane is :

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2x}{dy^2} = 0$

(c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

33. Given two vectors are $\hat{i} - \hat{j}$ and $\hat{i} + 2\hat{j}$ the unit vector coplanar with the two vectors and perpendicular to first is :

(a) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ (b) $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$

(c) $\pm\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$ (d) none of these

34. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$. The value of x are :

(a) $\left\{-\frac{2}{3}, 2\right\}$ (b) $\left\{\frac{1}{3}, 2\right\}$

(c) $\left\{\frac{2}{3}, 0\right\}$ (d) {2, 7}

35. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the co-ordinate planes. The length of a diagonal of the parallelopiped is :

(a) 7 unit (b) $\sqrt{38}$ unit

(c) $\sqrt{155}$ unit (d) none of these

36. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where :

(a) $ax_1+by_1+cz_1=0$

(b) $al+bm+cn=0$

(c) $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$

(d) $lx_1+my_1+nz_1=0$

37. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial, is :

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$

(c) $\frac{2}{25}$ (d) none of these

38. If A and B are two mutually exclusive events, then :

(a) $P(A)<P(\bar{B})$ (b) $P(A)>P(\bar{B})$

(c) $P(A)<P(B)$ (d) none of these

39. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is :

(a) $x=-1$ (b) $x=1$

(c) $x=-3/2$ (d) $x=3/2$

40. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals :

(a) 5 (b) 7

(c) 6 (d) 4

41. In a triangle ABC, $2ca\sin\frac{A-B+C}{2}$ is equal to :

(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$

(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

42. For $x\in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to :

(a) e (b) e^{-1}

(c) e^{-5} (d) e^5

43. The incentre of the triangle with vertices $(1,\sqrt{3})$, $(0,0)$ and $(2,0)$ is :

(a) $\left(1,\frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (d) $\left(1,\frac{1}{\sqrt{3}}\right)$

44. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then :

(a) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{b}=0$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

(d) $\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{c}\times\vec{a}=0$

45. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals :

(a) $128\,\omega$ (b) $-128\,\omega$

(c) $128\,\omega^2$ (d) $-128\,\omega^2$

46. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}=x+iy$, then :

(a) $x=3,\ y=1$ (b) $x=1,\ y=3$

(c) $x=0,\ y=3$ (d) $x=0,\ y=0$

47. $\sin^2\theta=\frac{4xy}{(x+y)^2}$ is true if and only if :

(a) $x+y\neq0$ (b) $x=y,\ x\neq0, y\neq0$

(c) $x=y$ (d) $x\neq0,\ y\neq0$

48. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at (0, 3), is :

(a) 4 unit (b) 3 unit

(c) $\sqrt{12}$ unit (d) $\frac{7}{2}$ unit

49. The probability of India winning a test match against West-Indies is 1/2 assuming independence from match to match. The probability that in a match series India's second win occurs at the third test is :

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

50. If $(\omega\neq1)$ is a cubic root of unity, then

$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$$ equals :

(a) zero (b) 1

(c) i (d) ω

51. A biased coin with probability p, $0<p<1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even, is 2/5, then p equals :

(a) 1/3 (b) 2/3
(c) 2/5 (d) 3/5

52. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is :

(a) $\frac{{}^7C_2 \times 5^5}{6^7}$ (b) $\frac{{}^7C_2 \times 5^5}{6^8}$

(c) $\frac{{}^7C_2 \times 5^5}{6^6}$ (d) none of these

53. Let $f(2) = 4$ and $f'(2) = 4$. Then $\lim_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$ is given by :

(a) 2 (b) – 2
(c) – 4 (d) 3

54. Three straight lines $2x + 11y - 5 = 0$, $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$:

(a) form a triangle
(b) are only concurrent
(c) are concurrent with one line bisecting the angle between the other two
(d) none of the above

55. A straight line through the point (2, 2) intersects the lines $\sqrt{3}x + y = 0$ and $\sqrt{3}x - y = 0$ at the points A and B. The equation to the line AB so that the triangle OAB is equilateral, is :

(a) $x - 2 = 0$ (b) $y - 2 = 0$

(c) $x + y - 4 = 0$ (d) none of these

56. The greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is :

(a) 10 unit (b) 15 unit
(c) 5 unit (d) none of these

57. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which make equal intercepts on the positive co-ordinate axes, is :

(a) $x + y = 2$ (b) $x + y = 2\sqrt{2}$
(c) $x + y = 4$ (d) $x + y = 8$

58. The equation of the ellipse whose foci are $(\pm 2, 0)$ and eccentricity is 1/2, is :

(a) $\frac{x^2}{12} + \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} + \frac{y^2}{12} = 1$

(c) $\frac{x^2}{16} + \frac{y^2}{8} = 1$ (d) none of these

59. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(a) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$

(b) $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(c) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

(d) $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

60. If the vectors $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is :

(a) $z\hat{i} - x\hat{k}$ (b) $\vec{0}$

(c) $y\hat{j}$ (d) $-z\hat{i} + x\hat{k}$

61. The centre of the circle given by $\vec{r} \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 15$ and $|\vec{r} - (\hat{j} + 2\hat{k})| = 4$ is :

(a) (0, 1, 2) (b) (1, 3, 4)
(c) (–1, 3, 4) (d) none of these

62. The value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is :

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

63. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is :

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) none of these

64. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$, then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to :

(a) 1 (b) –1
(c) zero (d) none of these

65. If $y = \sin^2\theta + \operatorname{cosec}^2\theta$, $\theta \neq 0$, then :

(a) $y = 0$ (b) $y \leq 2$
(c) $y \geq -2$ (d) $y > 2$

66. In a triangle ABC, $a = 4$, $b = 3$, $\angle A = 60°$, then c is the root of the equation :

(a) $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$
(c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$

67. In a ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then :

(a) a, c, b are in AP (b) a, b, c are in AP
(c) b, a, c are in AP (d) a, b, c are in GP

68. The equation $a\sin x + b\cos x = c$ where $|c| > \sqrt{a^2 + b^2}$ has :

(a) a unique solution
(b) infinite number of solutions
(c) no solution
(d) none of the above

69. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$ $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to :

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$

(c) $\frac{13}{18}$ (d) $-\frac{13}{18}$

70. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to :

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

71. $\sum_{n=0}^{\infty} \frac{(\log_e x)^n}{n!}$ is equal to :

(a) $\log_e x$ (b) x

(c) $\log_x e$ (d) none of these

72. $e^{(x-1) - \frac{1}{2}(x-1)^2 + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots}$ equal to :

(a) $\log(x-1)$ (b) $\log x$

(c) x (d) none of these

73. The coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$ is :

(a) 21 (b) 25

(c) 26 (d) none of these

74. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is :

(a) n (b) $n - 1$

(c) $n + 2$ (d) $n + 1$

75. The number of real roots of $3^{2x^2 - 7x + 7} = 9$ is :

(a) zero

(b) 2

(c) 1

(d) 4

ANSWERS

➡ PHYSICS AND CHEMISTRY

1.	(d)	2.	(c)	3.	(b)	4.	(a)	5.	(b)	6.	(a)	7.	(a)	8.	(a)
9.	(b)	10.	(c)	11.	(c)	12.	(b)	13.	(b)	14.	(a)	15.	(a)	16.	(d)
17.	(b)	18.	(b)	19.	(b)	20.	(d)	21.	(c)	22.	(b)	23.	(b)	24.	(c)
25.	(a)	26.	(a)	27.	(a)	28.	(c)	29.	(a)	30.	(c)	31.	(b)	32.	(a)
33.	(c)	34.	(b)	35.	(a)	36.	(a)	37.	(b)	38.	(b)	39.	(b)	40.	(c)
41.	(c)	42.	(b)	43.	(a)	44.	(c)	45.	(a)	46.	(c)	47.	(c)	48.	(b)
49.	(b)	50.	(b)	51.	(b)	52.	(c)	53.	(b)	54.	(d)	55.	(a)	56.	(d)
57.	(b)	58.	(c)	59.	(a)	60.	(a)	61.	(b)	62.	(d)	63.	(c)	64.	(d)
65.	(a)	66.	(b)	67.	(a)	68.	(b)	69.	(c)	70.	(b)	71.	(c)	72.	(a)
73.	(c)	74.	(a)	75.	(d)	76.	(d)	77.	(c)	78.	(b)	79.	(a)	80.	(a)
81.	(d)	82.	(c)	83.	(b)	84.	(a)	85.	(b)	86.	(b)	87.	(a)	88.	(b)
89.	(c)	90.	(a)	91.	(c)	92.	(c)	93.	(c)	94.	(b)	95.	(a)	96.	(a)
97.	(b)	98.	(b)	99.	(a)	100.	(c)	101.	(d)	102.	(b)	103.	(b)	104.	(d)
105.	(b)	106.	(c)	107.	(c)	108.	(a)	109.	(d)	110.	(a)	111.	(a)	112.	(d)
113.	(b)	114.	(c)	115.	(d)	116.	(c)	117.	(b)	118.	(c)	119.	(a)	120.	(c)
121.	(d)	122.	(c)	123.	(a)	124.	(c)	125.	(c)	126.	(d)	127.	(c)	128.	(a)
129.	(b)	130.	(a)	131.	(b)	132.	(d)	133.	(a)	134.	(b)	135.	(d)	136.	(c)
137.	(b)	138.	(b)	139.	(d)	140.	(a)	141.	(c)	142.	(d)	143.	(c)	144.	(c)
145.	(c)	146.	(a)	147.	(b)	148.	(a)	149.	(d)	150.	(c)				

➡ MATHEMATICS

1.	(b)	2.	(a)	3.	(b)	4.	(a)	5.	(b)	6.	(d)	7.	(d)	8.	(a)
9.	(b)	10.	(a)	11.	(c)	12.	(a)	13	(b)	14.	(a)	15.	(a)	16.	(b)
17.	(b)	18.	(a)	19.	(b)	20.	(d)	21.	(b)	22.	(b)	23.	(a)	24.	(b)
25.	(d)	26.	(a)	27.	(c)	28.	(b)	29.	(a)	30.	(a)	31.	(a)	32.	(a)
33.	(a)	34.	(a)	35.	(a)	36.	(b)	37.	(b)	38.	(a)	39.	(d)	40.	(b)
41.	(b)	42.	(c)	43.	(d)	44.	(b)	45.	(d)	46.	(d)	47.	(a)	48.	(a)
49.	(b)	50.	(a)	51.	(a)	52.	(b)	53.	(c)	54.	(c)	55.	(b)	56.	(b)
57.	(b)	58.	(b)	59.	(a)	60.	(a)	61.	(b)	62.	(c)	63.	(b)	64.	(a)
65.	(d)	66.	(a)	67.	(b)	68.	(c)	69.	(b)	70.	(d)	71.	(b)	72.	(c)
73.	(d)	74.	(d)	75.	(b)										