**IIT-JEE-Mathematics–1999**

Time : 3 hours                                                                    Max. Marks : 200
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**SECTION I**

**DIRECTIONS:** Select the most appropriate alternative A, B, C or D in questions 1-25.



 **2.** If x1, x2, x3 as well as y1, y2, y3 are in G.P. with the same common ratio, then the points (x1, y1), (x2, y2) and (x3, y3):
(A) lie on a straight line
(B) lie on the ellipse
(C) lie on a circle
(D) are vertices of a triangle

**3.** If the function *f* : [1, ∞) --> [1, -∞) is defined by *f* (x) = 2x(x–1), then *f*–1 (x) is : (A) (1/2)x(x-1)

(B) 1/2 (1+√(1+4 log\_2 x))
(C) 1/2(1-√(1+4 log2 x))
(D) not defined

**4.** The harmonic mean of the roots of the equation :
(5 + √2 x2 – (4 + √5) x + 8 + 2√5 = 0 is
(A) 2
(B) 4
(C) 6
(D) 8

**5.** The function *f* (x) = sin4 x + cos4 x increases if :
(A) 0 < x < π/8
(B) π/4 < x < 3π/8
(C) 3π/8 < x < 5π/8
(D) 5π/8 < x < 3π/4

**6.** The curve described parametrically by x = t2 + t + 1, y = t2– t + 1 represents :
(A) a pair of straight lines
(B) an ellipse
(C) a parabola
(D) a hyperbola

**7.** In a triangle PQR, ∠R = π/2. If tan (P/2) and tan (Q/2) are the roots of the equation ax2 + bx + c = 0 (a ≠ 0), then :
(A) a + b = c
(B) b + c = a
(C) a + c = b
(D) b = c

**8.** If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral ∫π/2 3π/2 [2 sinx. dx ] is∶
(A) -π
(B) 0
(C) -π/2
(D) π/2

**9.** Let a1, a2, ……, a10 be in A.P. and h1, h2, ……, h10  be in H.P. If a1 = h1 = 2 and a10 = h10 = 3, then a4 h7 is :
(A) 2
(B) 3
(C) 5
(D) 6



**11.** The number of real solutions of
tan–1 √(x(x+1)) + sin–1 √(x2+x+1)=π/2 is:
(A) zero
(B) one
(C) two
(D) infinite

**12.** Let P (a sec θ, b tan θ) and Q (a sec θ, b tan θ) where θ + θ = π/2, be two points on the hyperbola x2/a2 -y2/b2 = 1. If (h, k) is the point of intersection of the normals at P and Q, then K is equal to :
(A) (a2+b2)/a
(B) -((a2+b2)/a)
(C) (a2+b2)/b
(D) -((a2+b2)/b)

**13.** Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is 2x + y = 3, then the equation representing the pair of lines PQ and PR is :
(A) 3x2 – 3y2 + 8xy + 20x + 10y + 25 = 0
(B) 3x2 – 3y2 + 8xy – 20x – 10y + 25 = 0
(C) 3x2 – 3y2 + 8xy + 10x + 15y + 20 = 0
(D) 3x2 – 3y2 – 8xy – 10x – 15y – 20 = 0



**15.** The function f(x) = [x]2 – [x2] (where [y] is the greatest integer less than or

equal to y), is discontinuous at :
(A) all integers
(B) all integers except 0 and 1
(C) all integers except 0
(D) all integers except 1

**16.** If two distinct chords, drawn from the point (p, q) on the circle
x2+ y2=px + qy (where pq ≠ 0) are bisected by the x-axis, then :
(A) p2 = q2
(B) p2 = 8q2
(C) p2 <8q2
(D) p2 >8q2

**17.** The function f(x) = (x2 – 1)|x2 – 3x + 2|is NOT differentiable at:

(A) –1
(B) 0
(C) 1
(D) 2

**18.** If the roots of the equation x2 – 2ax + a2 + a – 3 = 0 are real and less than 3, then:
(A) a <2
(B) 2 < a < 3
(C) 3 <a < 4
(D) a >4

**19.** A solution of the differential equation (dx/dy)2-x (dx/dy) + y = 0 is :

(A) y = 2
(B) y = 2x
(B) y = 2x – 4
(D) y = 2x2 – 4

**20.** limx→0 (x tan2x - 2x tanx )/(1-cos2x )2  is :

(A) y = 2
(B) y = 2x
(C) y = 2x – 4
(D) y = 2x2 – 4



**22.** If in the expansion of (1 + x)n, the coefficients of x and x2 are 3 and –6 respectively, then m is :
(A) 6
(B) 9
(C) 12
(D) 24

**23.** ∫π/33π/4 dx/(1+cosx ) is equal to :

(A) 2
(B) –2
(C) 1/2
(D) -1/2

**24.** If x = 9 is the chord of contact of the hyperbola x2 – y2 = 0, then the equation of the corresponding pair of tangents is :
(A) 9x2 – 8y2 + 18x – 9 = 0
(B) 9x2 – 8y2 – 18x + 9 = 0
(C) 9x2 – 8y2 – 18x – 9 = 0
(D) 9x2 – 8y2 + 18x + 9 = 0

**25.** If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form 7m + 7n is divisible by 5 equals :
(A) 1/4
(B) 1/7
(C) 1/8
(D) 1/49

**DIRECTIONS :** Question numbers 26-35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions.

**26.** Let L1 be a straight line passing through the origin and L2 be the straight line x + y = 1. If the intercepts made by the circle x2 + y2 – x + 3y = 0 on L1 and L2 are equal, then which of the following equations can represent L1?
(A) x + y = 0
(B) x – y = 0
(B) x + 7y = 0
(D) x – 7y = 0



**28.** For a positive integer n, let a (n) = 1 + 1/2 + 1/3 + 1/4 +.……+ 1/((2n )-1). Then :
(A) a (100) ≤ 100
(B) a (100) > (100)
(C) a (200) ≤ 100
(D) a (200) > 100

**29.** The function f(x) = ∫-1xt (et-1)(t-1)(t-2)3 (t-3)5dt has a local minimum at x =

(A) 0
(B) 1
(C) 2
(D) 3

 **30.** On the ellipse 4x2 + 9y2 = 1, the points at which the tangents are parallel to the line 8x = 9y are :
(A) (2/5,1/5)
(B) (-2/5,1/5)
(C) (-2/5,-1/5)
(D) (2/5,-1/5)

**31.** The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true?
(A) p + m + c = 19/20
(B) p + m + c = 27/20
(C) pmc = 1/10
(D) pmc = 1/4

**32.** The differential equation representing the family of curves y2 = 2c (x + √c), where c is a positive parameter, is of :
(A) order 1
(B) order 2
(C) degree 3
(D) 10

**33.** Let S1, S2 … be squares such that for each n ≥ 1, the length of a side of Sn equals the length of a diagonal of Sn+1. If the length of a side of S1 is 10 cm, then for which of the following values of n is the area of Sn less than 1 sq. cm?
(A) 7
(B) 8
(C) 9
(D) 10

**34.** For which of the following values of m is the area of the region bounded by the curve y = x – x2 and he line y = mx equals 9/2?
(A) –4
(B) –2
(C) 2
(D) 4

**35.** For a positive integer n., let f\_n (θ)= (tan θ/2) (1 + secθ) (1 + sec2θ) (1 + sec 4θ) … (1 + sec 2nθ). Then

(A) f2 (π/16)=1 ;
(B) f3 (π/32)=1
(C) f4 (π/16)=1
(D) f5 (π/128)=1

**SECTION II**
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**Instructions**
There 12 questions in the section. Attempt ALL questions.
At the end of the anwers to a question, draw a horizontal line and start answer to the next question. The corresponding question number must be written in the left margin. Answer all parts of a question at one place only.
The use of Arabic numerals (0, 1, 2,…….9) only is allowed in answering the questions irrespective of the language in which you answer.
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**1.** For complex numbers z and q, prove that |z|2 w – |w|2 z = z – w if and only if z = w or z w-  = 1.

**2.** Let a, b, c d be real numbers in G.P. If u, v, w satisfy the system of equations
               u + 2v + 3 w = 6
              4u + 5v + 6 w = 12
              6u + 9v = 4
Then slow that the roots of the equation :
(1/u+1/v+1/w)x2 + [(b – c)2 + (c – a)2 + (d – b)2] x + u + v + w = 0
and 20x2 + 10 (a – d)2 x – 9 = 0 are reciprocals of each other.
**3.** Let n be any positive integer. Prove that :

    

**4.** Let ABC be a triangle having O and I as its circumcentre and incentre respectively. If R and r are the circumradius and the inradius, respectively, then prove that (IO)2 = R2 – 2Rr. Further show that the triangle BIO is a right-angled triangle if and only if b is the arithmetic mean of a and c.

**5.** Let T1, T2 be two tangents drawn from (–2, 0) onto the circle C : x2 + y2 = 1. Determine the circles touching C and having T1, T2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time.

**6.** Consider the family of circles x2 + y2 = r2, 2 < r < 5. If in the first quadrant, the common tangent to a circle of this family and the ellipse 4x2 + 25y2 = 100 meets the co-ordinate axes at A and B, then find the equation of the locus of the mid point of AB.



Find the area of the region in the third quadrant bounded by the curves x = –2y2 and y = f(x) lying on the left on the line 8x + 1 = 0.

**9.** Find the co-ordinates of all the P on the ellipse x2/a2 +y2/b2 = 1, for which the area of the triangle PON is maximum, where O denotes the origin and N, the foot of the perpendicular from O to the tangent at P.

**10.** A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve.

**11.** Eight players P1, P2, …… P8 play a knock-out tournament. It is known that whenever the players Pi and Pj play, the play Pi will win if i < j. Assuming that the players are paired at random in each round, what is the probability that the player P4 reaches the final?

