**IIT-JEE-Mathematics–1997**

**Time : Three Hours**                                                           **Max. Marks : 100**
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**1.** There are five sub-questions in this question. For answering each sub-question, four alternatives are given, and only one of them is correct. Indicate your answer for each sub-question by writing one of the letters A, B, C or D ONLY in the answer-book.

(i) If g (x)= ∫0xcos4  t dt, then *g (x + π)* equals :

(A) g (x)+ g (π)                         (B) g (x)- g (π)
(C) g (x)g (π)                            (D) (g (x))/(g (x))

(ii) If f (x)= x / sinx and g (x)= x  / tanx , where 0 < x ≤ 1, then in this interval :
(A) Both f (x) and g (x) are increasing functions.
(B) Both f (x) and g (x) are decreasing functions.
(C) f (x) is an increasing function.
(D) g (x) is an increasing function.

(iii) The parameter, on which the value of the determinant does not depend upon is :

                      

(A) a                                         (B) p
(C) d                                         (D) x

(iv) The graph of the function cos x (x + 2) – cos2 (x + 1) is:

(A) a straight line passing through (0, – sin2 1) with slope 2
(B) a straight line passing through (0, 0)
(C) a parabola with vertex (1, – sin2, 1)
(D) a straight line passing through the point (π/2,-sin2 1) and parallel to the x-axis.

(v)
                  

(A) 1 + √5                                                     (B) – 1+ √5
(C) – 1 + √2                              (D) 1 + √2

**2.** A tangent to the ellipse x2 + 4y2 = 4 meets the ellipse x2 + 2y2 = 6 at P and Q. Prove that the tangents at P and Q of the ellipse x2 + 2y2 = 6 are at right angles



**6.** The question contained FIVE incomplete statements. Complete these statements so that the resulting statements are correct. Write ONLY the answers in your answer-book in the order in which the statement are given below.
(i) The sum of all the real roots of the equations |x-2|2+|x-2|-2=0 is ….. ……… .

(ii) Let p and q be roots of the equations x2 – 2x + A = 0 and let r and s be the roots of the equation x2 – 18 + B = 0. If p < q < r < s are in arithmetic progression the A = ………….. and B = …………

 

    

10. Let z1 and z2 be roots of the equation z2 + pz + q = 0, where the co-efficients p and q may be complex numbers. Let A and B represent z1 and z2 in the complex plane. If  If  ∠AOB = α ≠ 0 and OA = OB, where O is the origin, prove that p2 = 4q cos2 (α/2).

11. The question contains FIVE incomplete statements. Complete these statements so that the resulting statements are correct. Write ONLY the answers in your answer-book in the order in which the statements are given below



12. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation x2 + px + q = 0 are real.

13. Let u (x) and v (x) satisfy the differential equations du/dx + p (x) u = f (x) and dv/dx + p (x) v = g (x), where p (x) and g (x) are continuous functions. If u (x1) > v (x1) for some x1 and f (x) > g (x) for all x > x1, prove that any point (x, y) where x > x1 does not satisfy the equations y = u (x) and y = v (x).

14. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side    of S. If a, b, c and d denote the lengths of the sides of the quadrilateral,
prove that 2 ≤  a2 + b2 + c2 + d2 ≤ 4.

15. Let f (x) = Maximum {x2, (1 – x)2, 2x (1 – x)}, where 0 ≤  x ≤  1. Determine the area of the region bounded by the curves y = f (x), x-axis, x = 0 and x = 1.

16. Prove that ∑k-1n-1 (n-k) cos(n/2), where n ≥ 3 is an integer.

17. Let C be any circle with centre (0,√2). Prove that at the most two rational points can be there on C.

(A rational point is a point both of whose coordinates are rational numbers.)