CODE P

INSTRUCTIONS

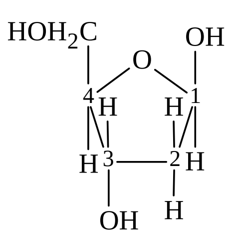
1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Block Ball Point Pen, Use of pencil is strictly prohibited.
2. The answer Sheet is kept inside this Test Booklet. When you are directed to pen the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Physics** and **Mathematics** having 30 questions in each part of equal weight age. Each question is allotted 4 (four) marks for each correct response.
6. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on **Side–1** and **Side–2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed to written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages (Page 21 – 23) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is P. Make sure that the CODE printed on Side–2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the answer sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

CHEMISTRY

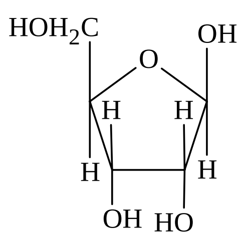
1. The presence or absence of hydroxyl group on which carbon atom of sugar differentiates RNA and DNA?
   (1) 1^{st} (2) 2^{nd}
   (3) 3^{rd} (4) 4^{th}

Key: (2)

Sol.:



DNA



RNA

2^{nd} carbon in DNA do not have OH group.

2. Among the following the maximum covalent character is shown by the compound
   (1) $FeCl_2$ (2) $SnCl_2$
   (3) $AlCl_3$ (4) $MgCl_2$

Key: (3)

Sol.: Higher the positive oxidation state higher will be the covalent character.

3. Which of the following statement is wrong?
   (1) The stability of hydride increases from NH_3 to BiH_3 in group 15 of the periodic table.
   (2) Nitrogen cannot from $d\pi$ - $p\pi$ bond.
   (3) Single N – N bond is weaker than the single P – P bond.
   (4) N_2O_4 has two resonance structures.

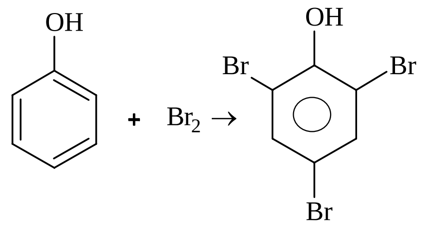
Key: (1)

Sol.: As we move down the group, tendency to from covalent bond with small H decreases hence M-H bond enthalpy decreases.

4. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :
   (1) 2-Bromophenol
   (2) 3-Bromophenol
   (3) 4-Bromophenol
   (4) 2, 4, 6-Tribromophenol

Key: (4)

Sol.: $5KBr + KBrO_3 + 3H_2O \rightarrow 3Br_2 + 6KOH$



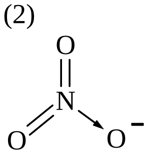
5. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution?
(1) 0.100 (2) 0.190
(3) 0.086 (4)` 0.050

Key: (3)

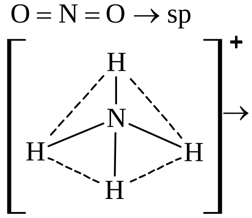
Sol.: 5.2 mole of CH_3OH in 1000 gram water i.e. in $\frac{1000}{18}$ mole H_2O mole fraction of CH_3OH = $\frac{5.2}{5.2+\frac{1000}{18}} = 0.086$.

6. The hybridization of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :
(1) sp, sp^2, sp^3 (2) sp^2, sp, sp^3
(3) sp, sp^3, sp^2 (4) sp^2, sp^3, sp

Key: (2)

Sol.:  $\rightarrow sp^2$

$O = \overset{+}{N} = O \rightarrow sp$

 $\rightarrow sp^3$

7. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at –6°C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})
(1) 804.32 g (2) 204.30 g
(3) 400.00 g (4) 304.60 g

Key: (1)

Sol.: ΔT_f = kf.m = 0 – (- 6) = 1.86 m

m = $\frac{6}{1.86}$ i.e., = $\frac{6}{1.86}$ mole in 1 kg.

There for $\frac{6}{1.86}\times 4$ mole in 4 kg.

Wt = $\frac{6}{1.86}\times 4\times 62$ =804.32gram.

8. The reduction potential of hydrogen half-cell will be negative if :
(1) $p(H_2)$ = 1 atom and $[H^+]$ = 2.0 M
(2) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M
(3) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M
(4) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M

Key: (3)

Sol.: $2H^+ + 2e^- \longrightarrow H_2\,(g)$

$$E_{H^+/H_2} = E^\circ_{H^+/H_2} - \frac{0.0059}{2}\log\frac{P_{H_2}}{|H^+|^2}$$

$$= 0 - \frac{0.59}{2}\log\frac{P_{H_2}}{|H^+|^2}$$

For the negative value of E_{H^+/H_2}

By $\frac{P_{H_2}}{\left[H^+\right]^2}$ should be +ve i.e $P_{H_2} > |H^+|$

9. Which of the following reagents may be used to distinguish between phenol and benzoic acid?
(1) Aqueous NaOH (2) Tollen's reagent
(3) Molisch reagent (4) Neutral $FeCl_3$

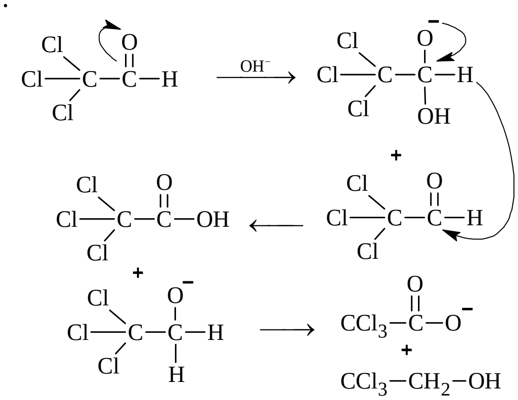
Key: (4)

Sol.: $FeCl_3$ forms violet complex with phenol whereas it forms buff coloured ppt with Benzoic Acid.

10. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :
(1) 2, 2, 2-Trichloroethanol
(2) Trichloromethanol
(3) 2, 2, 2-Trichloropropanol
(4) Chloroform

Key: (1)

Sol.:



11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?
(1) $Al_2O_3 < MgO < Na_2O < K_2O$

(2) $MgO < K_2O < Al_2O_3 < Na_2O$
(3) $Na_2O < K_2O < MgO < Al_2O_3$
(4) $K_2O < Na_2O < Al_2O_3 < MgO$

Key: (1)

Sol.: Metallic property increases down the group and decreases across a period when moved from left to right.

12. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at:
(1) 1035 nm (2) 325 nm
(3) 743 nm (4) 518 nm

Key: (3)

Sol.: Energy of absorbed photon = Sum of the energies of emitted photons

$$\frac{hc}{355\times10^{-9}} = \frac{hc}{680\times10^{-9}} + \frac{hc}{x}$$

$\Rightarrow$ x = 742.77×10^{-9}m i.e. 743 nm.

13. Which of the following statements regarding sulphur is incorrect?
(1) S_2 molecule is paramagnetic.
(2) The vapour at 200°C consists mostly of S_8 rings.
(3) At 600°C the gas mainly consists of S_2 molecules.
(4) The oxidation state of sulphur is never less than +4 in its compounds.

Key: (4)

Sol.: Oxidation state of sulphur ranges between –2 to +6 in different compounds.

14. The entropy change involved in the isothermal reversible expansion of 2 mole of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is:
(1) 38.3 J mol^{-1} K^{-1} (2) 35.8 J mol^{-1} K^{-1}
(2) 32.3 J mol^{-1} K^{-1} (4) 42.3 J mol^{-1} K^{-1}

Key: (1)

Sol.: $\Delta s = 2.303nR \log \frac{v_f}{v_i}$

$= 2.303\times 2 \times 8.314 \log \frac{100}{10}$

$= 38.294 \approx 38.3$ J $mol^{-1}K^{-1}$.

15. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?
(1) The complex involves d^2sp^3 hybridisation and is octahedral in shape.
(2) The complex is paramagnetic.
(3) The complex is an outer orbital complex
(4) The complex gives white precipitate with silver nitrate solution.

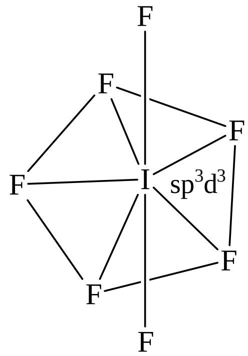
Key: (3)

Sol.: It is an inner orbital complex as the d-orbital involved in hybridization belongs to penultimate shell.

16. The structure of IF_7 is
(1) square pyramid
(2) trigonal bipyramid
(3) octahedral
(4) pentagonal bipyramid

Key: (4)

Sol.: pentagonal bipyramidal shape.



17. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about :
(1) 10 times (2) 24 times
(3) 32 times (4) 64 times

Key: (3)

Sol.: rate of reactions increases by
(temp. coef.)$^{\text{no. of interval of 10°C}}$
$=2^5 = 32$ times.

18. The strongest acid amongst the following compounds is :
(1) CH_3COOH
(2) HCOOH
(3) $CH_3CH_2CH(Cl)CO_2H$
(4) $ClCH_2CH_2CH_2COOH$

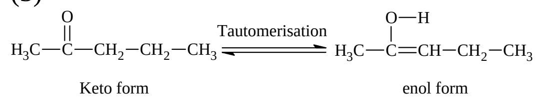
Key: (3)

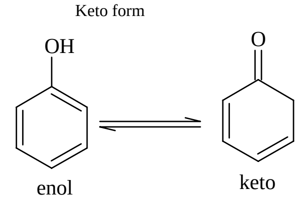
Sol.: Presence of one –I effect chlorine at α-carbon increases the acid strength significantly.

19. Identify the compound that exhibits tautomerism :
(1) 2-butene (2) Lactic acid
(3) 2-Pentanone (4) Phenol

Key: (3)

Sol.:





20. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :
(1) 1.8 atm (2) 3 atm
(3) 0.3 atm (4) 0.18 atm

Key: (1)

Sol.: $$CO_{2(g)} + C_{(g)} \xrightarrow{1000K} 2CO_{(g)}$$

initial pressure 0.5atm 0

final pressure (0.5–x)atm 2x atm

total pressure at equil = $p_{CO_2} + p_{CO}$

=(0.5 – x) + 2x = 0.8 atm (Given)

$\Rightarrow$ x = 0.3 atm.

$\therefore$ Equil const $K_p = \dfrac{(p_{CO})^2}{p_{CO_2}}$

$= \dfrac{(0.6)^2}{0.2}$ = 1.8 atm.

21. In context of the lanthanoids, which of the following statements is not correct?
(1) There is a gradual decrease in the radii of the members with increasing atomic number in the series.
(2) All the members exhibit +3 oxidation state.
(3) Because of similar properties the separation of lanthanoids is not easy.
(4) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

Key: (4)

Sol.: Lanthanoids exhibit +3 oxidation sate without an exception.

22. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because
(1) a and b for Cl_2 > a and b for C_2H_6
(2) a and b for Cl_2 < a and b for C_2H_6
(3) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6
(4) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

Key: (4)

Sol.: Compressible gases have greater force of attraction and hence value of 'a' should be greater and reduced volume 'b' should be less.

23. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :
(1) 1.82 BM (2) 5.46 BM
(3) 2.82 BM (4) 1.41 BM

Key: (3)

Sol.: Cl^- is a weak field ligand and therefore d^8 ion will have two unpaired electron.
$\mu = \sqrt{n(n+2)} = \sqrt{2\times 4} = \sqrt{8} = 2.82$ B.M.

24. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :
(1) A_2B (2) AB_2
(3) A_2B_3 (4) A_2B_5

Key: (4)

Sol.: No. of atoms in the corners (A) = $8 \times \frac{1}{8} = 1$

No. of atom at face centres (B) = $5 \times \frac{1}{2} = 2.5$

Formula $AB_{2.5}$ i.e. A_2B_5

25. The outer electron configuration of Gd (Atomic No. : 64) is :
(1) $4f^3\, 5d^5\, 6s^2$ (2) $4f^8\, 5d^0\, 6s^2$
(3) $4f^4\, 5d^4\, 6s^2$ (4) $4f^7\, 5d^1\, 6s^2$

Key: (4)

Sol.: The configuration is $4f^7\, 5d^1\, 6s^2$.

26. Boron cannot form which one of the following anions?
(1) BF_6^{3-} (2) BH_4^-
(3) $B(OH)_4^-$ (4) BO_2^-

Key: (1)

Sol.: Boron's maximum covalency is 4.

27. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :
(1) two ethylenic double bonds
(2) a vinyl group
(3) an isopropyl group
(4) an actylenic triple bond

Key: (2)

Sol.: Compound must have $-\underset{|}{C} = CH_2$ group in order to give formaldehyde as one of the products.

28. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :
(1) Diethyl ether
(2) 2-Butanone
(3) Ethyl chloride
(4) Ethyl ethanoate

Key: (4)

Sol.: $CH_3CH_2O^-Na^+ + CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \longrightarrow$

$CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-O-CH_2CH_3 + NaCl$

Nucleophilic acyl substitution.

29. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression

(1) $\alpha = \dfrac{i-1}{(x+y-1)}$

(2) $\alpha = \dfrac{i-1}{x+y+1}$

(3) $\alpha = \dfrac{x+y-1}{i-1}$

(4) $\alpha = \dfrac{x+y+1}{i-1}$

Key: (1)

Sol.: $A_xB_y \longrightarrow xA^{y+} + yB^{x-}$

$1-\alpha \qquad x\alpha \qquad y\alpha$

Van't Hoff factor 'i' = $1 - \alpha + x\alpha + y\alpha$

$\therefore \quad \alpha = \dfrac{i-1}{(x+y-1)}$

30. Silver Mirror test is given by which one of the following compounds?

(1) Acetaldehyde (2) Acetone

(3) Formaldehyde (4) Benzophenone

Key: (1) or (3)

Sol.: $R-CHO + Ag(NH_3)_2^+ + OH^- \rightarrow$

$RCOO^- + Ag + NH_4^+$.

PHYSICS

31. 100 g of water is heated from 30ºC to 50ºC. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K)

(1) 4.2 kJ (2) 8.4 kJ

(3) 84 kJ (4) 2.1 kJ.

Key. (2)

Sol. $\Delta U = 0.1 \times 4184 \times 20 \cong 8.4$ kJ.

$\therefore$ (2).

32. The half life of a radioactive substance is 20 minutes. The approximate time internal ($t_2 - t_1$) between the time t_2 when $\dfrac{2}{3}$ of it had decayed is

(1) 7 min (2) 14 min

(3) 20 min (4) 28 min.

Key. (3)

Sol. $\dfrac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = 2$

$\Rightarrow \; t_2 - t_1 = \dfrac{\ell n2}{\lambda} = T_{\frac{1}{2}} = 20$ min.

$\therefore$ (3).

33. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\dfrac{A_1}{A_2}\right)$ is

(1) $\dfrac{M}{M+m}$ (2) $\dfrac{M+m}{M}$

(3) $\left(\dfrac{M}{M+m}\right)^{1/2}$ (4) $\left(\dfrac{M+m}{M}\right)^{1/2}$.

Key. (4)

Sol. COM $\Rightarrow \; MA_1\sqrt{\dfrac{k}{M}} = (M+m)V$

Also $V = A_2\sqrt{\dfrac{k}{M+m}}$.

$\therefore$ (4).

34. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(1) 12.1 eV (2) 36.3 eV

(3) 108.8 eV (4) 122.4 eV.

Key. (3)

Sol. $\Delta U = 13.6(3)^2\left(\dfrac{1}{1^2} - \dfrac{1}{3^2}\right) = 108.8$ eV

$\therefore$ (3).

35. The transverse displacement y (x, t) of a wave on a string is given by

$y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$

This represents a

(1) wave moving in +x direction with speed $\sqrt{\dfrac{a}{b}}$

(2) wave moving in +x direction with speed $\sqrt{\dfrac{b}{a}}$

(3) standing wave of frequency $\sqrt{b}$

(4) standing wave of frequency $\dfrac{1}{\sqrt{b}}$.

Key. (2)

Sol. $y(x,\,t) = e^{-(\sqrt{a}x+\sqrt{b}t)^2}$

$\therefore$ (2).

36. A resistor R and 2μF capacitor in series in connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$)
(1) $1.3 \times 10^4\ \Omega$ (2) $1.7 \times 10^5\ \Omega$
(3) $2.7 \times 10^6\ \Omega$ (4) $3.3 \times 10^7\ \Omega$.

Key. (3)

Sol. $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$

$\Rightarrow\ 120 = 200\left(1 - e^{\frac{-5}{R\times2\times10^{-6}}}\right)$

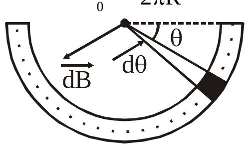
$\Rightarrow\ R = 2.7\times10^6\,\Omega$.

$\therefore$ (3)

37. A current I flows in a infinitely long wire with cross section in the form of a semi–circular ring of radius R. The magnitude of the magnetic induction along its axis is
(1) $\frac{\mu_0 I}{\pi^2 R}$ (2) $\frac{\mu_0 I}{2\pi^2 R}$
(3) $\frac{\mu_0 I}{2\pi R}$ (4) $\frac{\mu_0 I}{4\pi R}$.

Key. (1)

Sol. $B = \int dB\sin\theta = \int_0^{\pi} \frac{\mu_0\left(\frac{I}{\pi}\cdot d\theta\right)}{2\pi R}\sin\theta = \frac{\mu_0 I}{\pi^2 R}$.



$\therefore$ (1).

38. A Carnot engine operating between temperatures T_1 and T_2 has efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively :
(1) 372 K and 310 K
(2) 372 K and 330 K
(3) 330 K and 268 K
(4) 310 K and 248 K.

Key. (1)

Sol. $\eta = 1 - \frac{T_2}{T_1}$

$\therefore$ (1).

39. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by
$$\frac{dv}{dt} = -2.5\sqrt{v}$$
where v is the instantaneous speed. The time taken by the object, to come to rest, would be
(1) 1 s (2) 2 s
(3) 4 s (4) 8 s.

Key. (2)

Sol. $\int_{6.25}^{0} \frac{d\theta}{\sqrt{v}} = -2.5\int_0^t dt$

$\Rightarrow$ t = 2 s

$\therefore$ (2)

40. The electrostatic potential inside a charged spherical ball is given by $\phi = a r^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is
(1) $-24\pi a\varepsilon_0 r$ (2) $-6 a\varepsilon_0 r$
(3) $-24\pi a\varepsilon_0$ (4) $-6 a\varepsilon_0$.

Key. (4)

Sol. $\phi = ar^2 + b \ \Rightarrow\ E = -2ar$

Now, $\oint_{\text{sphere}} \vec{E}\cdot d\vec{s} = \frac{q_{encl}}{\varepsilon_0}$

$$-2ar\cdot 4\pi r^2 = \frac{\rho\cdot\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$\Rightarrow\ \rho = -6a\varepsilon_0$.

$\therefore$ (4).

41. A car is fitted with a convex side–view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is
(1) $\frac{1}{10}$ m/s (2) $\frac{1}{15}$ m/s
(3) 10 m/s (D) 15 m/s.

Key. (2)

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow\ -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$

$\Rightarrow\ \frac{dv}{dt} = 15\left(\frac{280}{15\times280}\right)^2 \cong \frac{1}{15}$ m/s

$\therefore$ (2).

42. If a wire is stretched to make it 0.1% longer, its resistance will
(1) increase by 0.05%
(2) increase by 0.2%
(3) decrease by 0.2%
(4) decrease by 0.05%.

Key. (2)

Sol. $R = \rho\frac{\ell}{A} = \frac{\rho\ell^2}{(\text{Volume})}$

$\Rightarrow\ R \propto \ell^2$

$\therefore \quad \frac{\Delta R}{R} = 2\frac{\Delta \ell}{\ell}$

$\therefore \quad$ (2).

43. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(1) $\frac{(T_1 + T_2 + T_3)}{3}$

(2) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(3) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^3}{n_1T_1 + n_2T_2 + n_3T_3}$

(4) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3T_3^3}{n_1T_1 + n_2T_2 + n_3T_3}$.

Key. (2)

Sol. Number of moles of first gas $= \frac{n_1}{N_A}$

Number of moles of second gas $= \frac{n_2}{N_A}$

Number of moles of thirst gas $= \frac{n_3}{N_A}$

If no loss of energy then

$P_1V_1 + P_2V_2 + P_3V_3 = PV$

$\frac{n_1}{N_A}RT_1 + \frac{n_2}{N_A}RT_2 + \frac{n_3}{N_A}RT_3$

$= \frac{n_1 + n_2 + n_3}{N_A}RT_{mix}$

$T_{mix} = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$.

$\therefore \quad$ (2).

44. Two identical charged spheres suspended from a common point by two massless strings of length ℓ are initially a distance d ($d << \ell$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

(1) $v \propto x^{\frac{1}{2}}$ (2) $v \propto x^{-1}$

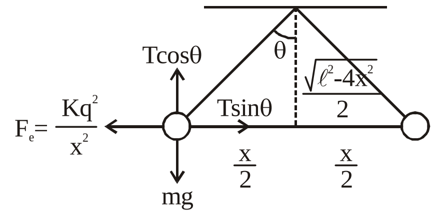
(3) $v \propto x^{-\frac{1}{2}}$ (4) $v \propto x$.

Key. (3)

Sol. $T\sin\theta = \frac{Kq^2}{x^2}$ …(i)

$T\cos\theta = mg$ …(ii)

$\text{Tan}\theta = \frac{x}{\sqrt{\ell^2 - 4x^2}} = \frac{x}{\ell}$



$\frac{x}{\ell} = \frac{kq^2}{x^2mg}$

$x^3 = \frac{kq^2\ell}{mg}$ …(i)

$x^3 \propto q^2$

$3x^2\frac{dx}{dt} \propto 2q\frac{dq}{dt}$

$x^2 \cdot v \propto q$

$v \propto x^{-\frac{1}{2}}$.

$\therefore \quad$ (3)

45. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

(1) 4 π mJ (2) 0.2 π mJ

(3) 2 π mJ (4) 0.4 π mJ.

Key. (4)

Sol. W = (surface energy)$_{final}$ – (surface energy)$_{initial}$

$W = T \times 4\pi\left[\left(5\times10^{-4}\right) - (3\times10^{-2}\right]\times 2$

$= 4\pi \times 0.03 \times 16 \times 10^{-4} \times 2$

$= 4\pi \times 0.48 \times 10^{-4} \times 2$

$= 1.92\pi \times 10^{-4} \times 2$

$= 3.94\pi \times 10^{-4} = 0.394\ \pi$ mJ $\approx 0.4\pi$ mJ.

$\therefore \quad$ (4).

46. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is

(1) $\pi\sqrt{LC}$ (2) $\frac{\pi}{4}\sqrt{LC}$

(3) $2\pi\sqrt{LC}$ (4) $\sqrt{LC}$.

Key. (2)

Sol. $\frac{q_0^2}{2C} = \frac{q^2}{2C} + \frac{Li^2}{2}$

differentiating w.r.t. t

$\frac{di}{dt} = -\frac{q}{LC}$

$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$

Comparing $\frac{d^2x}{dt^2} = -\omega^2 x$

$$\omega = \frac{1}{\sqrt{LC}}$$

So, $q = q_0 \cos\omega t$ ($\because$ at $t = 0$, $q = q_0$)

For half energy $q = \frac{q_0}{\sqrt{2}}$

So, $\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$

$$\omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}.$$

$\therefore$ (2).

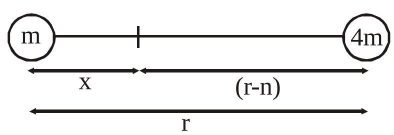
47. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(1) zero (2) $-\frac{4Gm}{r}$

(3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$.

Key. (4)

Sol.



Let gravitational field at P is zero

$$\frac{Gm}{x^2} = \frac{G \times 4m}{(r = n)^2}$$

$$x = \frac{r}{4}$$

Now potential at P

$$V_P - = \frac{Gm}{x} - \frac{G(4m)}{(r-n)}$$

$$= -\frac{Gm}{(r/3)} - \frac{4Gm}{(2r/3)}$$

$$= -\frac{9Gm}{r}.$$

$\therefore$ (4).

48. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc of reach its other end. During the journey of the insect, the angular speed of the disc

(1) remains unchanged

(2) continuously decreases

(3) continuously increases

(4) first increases and then decreases.

Key. (4)

Sol. Angular momentum $L = I\omega$

$L = mr^2.\omega$

Since r first decrease then increases

So due to conservation of angular momentum L first increases then decreases.

49. Let the x – z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(1) 30º (2) 45º

(3) 60º (4) 75º.

Key. (2)

Sol. Angle of incidence with Z direction (normal)

$$\cos\alpha = \frac{10}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + (10)^2}} = \frac{1}{2}$$

$\alpha = 60º$.

So, $\mu_1 \sin\alpha = \mu_2 \sin\beta$

$$\sqrt{2} \times \sin 60 = \sqrt{3}\sin\beta$$

$$\beta = 45º.$$

$\therefore$ (2)

50. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x–axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is (X_0 + A), the phase difference between their motion is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$.

Key. (2)

51. **Direction :**
The question has a paragraph followed by two statements, **Statement – 1** and **Statement – 2**. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane–convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement – 1 :
When light reflects from the air–glass plate interface, the reflected wave surfers a phase change of π.

Statement – 2 :
The centre of the interference pattern is dark.

(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

Key. (2)

52. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(1) $\frac{(\gamma-1)}{2(\gamma+2)R}Mv^2K$

(2) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$

(3) $\frac{\gamma Mv^2}{2R}K$

(4) $\frac{(\gamma-1)}{2R}Mv^2K$.

Key. (4)

Sol. $\frac{1}{2}Mv^2 = \frac{R}{\gamma-1}\Delta T$

$\Rightarrow \Delta T = \frac{(\gamma-1)}{2R}Mv^2\,K$.

$\therefore$ (4)

53. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 9 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is

(1) 0.52 cm (2) 0.052 cm
(3) 0.026 cm (4) 0.005 cm.

Key. (2)

Sol. d = MSR + CSR

$= 0 + 52\times\frac{1}{100} = 0.52$ mm.

$\therefore$ (2)

54. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(1) 1 mV (2) 0.75 mV
(3) 0.50 mV (4) 0.15 mV.

Key. (4)

Sol. $\varepsilon_{ind} = Bv\ell$

$= 5\times10^{-5}\times1.50\times2 = 0.15$ mV.

$\therefore$ (4)

55. **Direction :**
The question has **Statement – 1** and **Statement – 2**. Of the four choices given after the statements, choose the one that describes the two statements.

Statement – 1 :
Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement – 2 :
The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

Key. (2)

Sol.

56. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(1) $\frac{3}{2}g$ (2) g

(3) $\frac{2}{3}g$ (4) $\frac{g}{3}$.

Key. (3)

Sol. Equations of motion are

$mg - T = ma$ …(i)

and $T \cdot R = \frac{1}{2}mR^2\alpha$ …(ii)

and $a = R\alpha$ …(iii)

Solving $a = \frac{2}{3}g$.

$\therefore$ (3)

57. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$

(3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$.

Key. (2)

Sol. $A = \pi R_{max}^2 = \frac{\pi v^4}{g^2}$.

$\therefore$ (2)

58. **Direction :**

The question has **Statement – 1** and **Statement – 2**. Of the four choices given after the statements, choose the one that describes the two statements.

Statement – 1 :

A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement – 2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement – 1 is True, Statement – 2 is False.

(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.

(4) Statement – 1 is False, Statement – 2 is True.

Key. (4)

Sol. $K_{max} = hv - w$

and $K_{max} = eV_S$.

$\therefore$ (4)

59. A pulley of radius 2 m is rotated about its axis by a force F = $(20t - 5t^2)$ Newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations make by the pulley before its direction of motion if reversed, is

(1) less than 3

(2) more than 3 but less than 6

(3) more than 6 but less than 9

(4) more than 9.

Key. (2)

Sol. $\alpha = \frac{\tau}{I} = 4t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 4t - t^2$

$\Rightarrow \omega = 2t^2 - \frac{t^3}{3}$

ω is zero at t = 0s and t = 6s

Now $\frac{d\theta}{dt} = \omega = 2t^2 - \frac{t^3}{3}$

$\Rightarrow \theta = \frac{2}{3}t^3 - \frac{t^4}{12}$

θ at t = 6s = 36 rad

$\therefore$ number of rotations $= \frac{36}{2\pi} < 6$.

$\therefore$ (2).

60. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. the water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(1) 5.0×10^{-3} m (2) 7.5×10^{-3} m

(3) 9.6×10^{-3} m (4) 3.6×10^{-3} m.

Key. (4)

Sol. $A_1v_1 = A_2v_2$

and $v_2^2 = v_1^2 + 2gh$.

$\therefore$ (4).

MATHEMATICS

61. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that:

(1) $\beta \in (0, 1)$ (2) $\beta \in (-1, 0)$
(3) $|\beta| = 1$ (4) $\beta \in (1, \infty)$

Key: (4)

Sol.: Let roots be 1 + ia and 1 - ia

So $(1 + ia) + (1 - ia) = -\alpha$

and $(1 + ia)(1 - ia) = \beta$

$\Rightarrow \beta = 1 + a^2$

$\Rightarrow \beta \in (1, \infty)$

62. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

(1) $\pi \log 2$ (2) $\frac{\pi}{8}\log 2$
(3) $\frac{\pi}{2}\log 2$ (4) $\log 2$

Key: (1)

Sol.: $I = \int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$I = \int_0^{\pi/4} 8\log(1+\tan\theta)\,d\theta$

$I = 8\int_0^{\pi/4} \log(1+\tan\left(\frac{\pi}{4}-\theta\right))d\theta$

$= 8\int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$

$= 8\left[\int_0^{\pi/4}\left(\log 2 - \log(1+\tan\theta)\right)d\theta\right.$

$I = 4\int_0^{\pi/4} \log 2\, d\theta = \pi \log 2$

63. $\frac{d^2x}{dy^2}$ equals

(1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
(3) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Key: (4)

Sol.: $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\left(\frac{dy}{dx}\right)^{-1}\right)$

$= \frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^{-1}\right)\left(\frac{dy}{dx}\right)^{-1}$

$= -\left(\frac{d^2y}{dx^2}\right) \cdot \left(\frac{dy}{dx}\right)^{-3}$

64. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

(1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$
(3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

Key (2)

Sol.: $\frac{dV(t)}{dt} = -k(T-t)$

$V(t) = \frac{k(T-t)^2}{2} + c$

at t = 0, V(t) = I $\Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

$V(T) = I + \frac{k}{2}(T^2 - 2T^2)$

$= I - \frac{K}{2}T^2$

65. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(1) 144 (2) - 132
(3) - 144 (4) 132

Key: (3)

Sol.: $(1 - x + x^2 + x^3)^6 = (1 - x)^6 (1 - x^2)^6$

$= (1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6)$

$x(1 - 6x^2 + 15x^4 - 20x^6 + 15x^8 - 6x^{10} + x^{12})$

coefficient of x^7 = (-6) (-20) + (-20) (15)

+ (-6) (-6)

= 120 - 300 + 36

= -144

66. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\ \sin t\, dt$. Then f has

(1) local maximum at π and 2π

(2) local minimum at π and 2π

(3) local minimum at π and local maximum at 2π

(4) local maximum at π and local minimum at 2π

Key: (4)

Sol.: $f(x) = \int_0^x \sqrt{t}\sin t\, dt$

$f'(x) = \sqrt{x}\ \sin x$

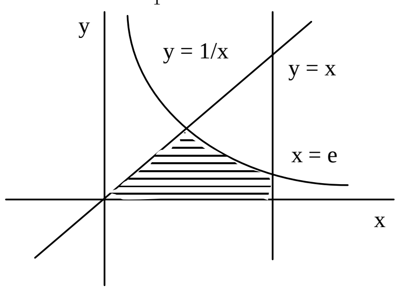
+	−	+	
0	π	2π	$5\pi/2$

f(x) has local maximum at π and local minima at 2π

67. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is
(1) 1/2 square units (2) 1 square units
(3) 3/2 square units (4) 5/2 square units

Key: (3)

Sol.: Area = $1/2 + \int_1^e \frac{1}{x}dx$

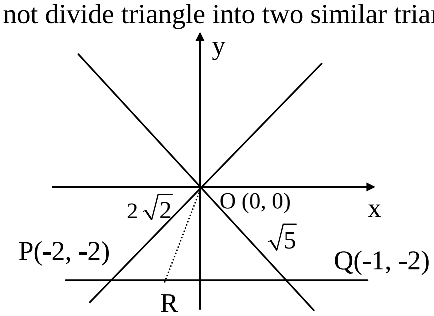


$= \frac{1}{2} + \ln|x|\Big|_1^e = \frac{3}{2}$

68. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
Statement-1:
The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$
Statement-2:
In any triangle bisector of an angle divides the triangle into two similar triangles.
(1) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key: (3)

Sol.: In ΔOPQ angle bisector of O divides PQ in the ratio of OP : OQ which is $2\sqrt{2} : \sqrt{5}$ but it does not divide triangle into two similar triangles.



69. The values of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$ is

continuous for all x in R, are
(1) $p = \frac{1}{2}, q = -\frac{3}{2}$ (2) $p = \frac{5}{2}, q = \frac{1}{2}$
(3) $p = -\frac{3}{2}, q = \frac{1}{2}$ (4) $p = \frac{1}{2}, q = \frac{3}{2}$

Key: (3)

Sol.: $$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} \frac{\sin(p+1)x + \sin x}{x} = p + 2$

$\lim_{x \to 0^+} f(x) = \frac{1}{2} \Rightarrow p + 2 = q = \frac{1}{2}$

$\Rightarrow p = -\frac{3}{2}, q = \frac{1}{2}$

70. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals
(1) 2/3 (2) 3/2
(3) 2/5 (4) 5/3

Key: (1)

Sol.: $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$
$x + 2y + 3z = 4$
Angle between the line and plane will be
$\theta = \sin^{-1}\left(\frac{1.1 + 2.2 + \lambda.3}{\sqrt{1+4+\lambda^2}\sqrt{1+4+\lambda}}\right) = \sin^{-1}\left(\frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}\right)$

$= \cos^{-1}\left(\sqrt{1 - \frac{(5+3\lambda)^2}{14(5+\lambda^2)}}\right) = \cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$

(given)
$\Rightarrow \lambda = 2/3$.

71. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(1) $(-\infty, \infty)$ (2) $(0, \infty)$
(3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

Key: (3)

Sol.: $f(x) = \dfrac{1}{\sqrt{|x|-x}}$

f(x) is define if |x| - x > 0

$\Rightarrow |x| > x$

$\Rightarrow x < 0$

So domain of f(x) is $(-\infty, 0)$.

72. The shortest distance between line y - x = 1 and curve $x = y^2$ is

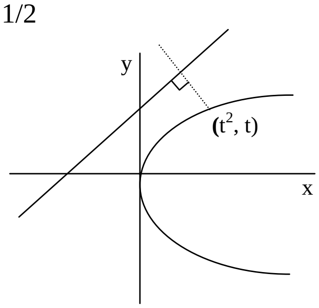
(1) $\dfrac{\sqrt{3}}{4}$ (2) $\dfrac{3\sqrt{2}}{8}$

(3) $\dfrac{8}{3\sqrt{2}}$ (4) $\dfrac{4}{\sqrt{3}}$

Key: (2)

Sol.: Shortest distance between two curve occurred along the common normal, so -2t = -1

$\Rightarrow t = 1/2$



So shortest distance between them is $\dfrac{3\sqrt{2}}{8}$

73. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after.

(1) 18 months (2) 19 months

(3) 20 months (4) 21 months

Key: (4)

Sol.: Let it happened after m months

$2 \times 300 + \dfrac{m-3}{2}$ (2 × 240 + (m - 4) × 40))

= 11040

$\Rightarrow m^2 + 5m - 546 = 0$

$\Rightarrow (m + 26)(m - 21) = 0 \Rightarrow m = 21.$

74. Consider the following statements

P: Suman is brilliant

Q : Suman is rich

R: Suman is honest

The negation of the statement Suman is brilliant and dishonest if and only if Suman is rich can be expressed as

(1) $\sim P \wedge (Q \leftrightarrow \sim R)$ (2) $\sim (Q \leftrightarrow (P \wedge \sim R))$

(3) $\sim Q \leftrightarrow \sim P \wedge R$ (4) $\sim (P \wedge \sim R) \leftrightarrow Q$

Key: (2)

Sol.: Suman is brilliant and dishonest if and only if Suman is rich is expressed as

$Q \leftrightarrow (P \wedge \sim R)$

Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$

75. If ω ($\neq 1$) is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals:

(1) (0, 1) (2) (1, 1)

(3) (1, 0) (4) (-1, 1)

Key: (2)

Sol.: $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$\Rightarrow A = 1, B = 1.$

76. If $\vec{a} = \dfrac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \dfrac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is

(1) - 5 (2) -3

(3) 5 (4) 3

Key: (1)

Sol.: $\left(2\bar{a} - \bar{b}\right).\left(\left(\bar{a} \times \bar{b}\right) \times \left(\bar{a} + 2\bar{b}\right)\right)$

$= \left(2\bar{a} - \bar{b}\right).\left(\left(\bar{a} \times \bar{b}\right) \times \bar{a} + 2\left(\bar{a} \times \bar{b}\right) \times \bar{b}\right)$

$= \left(2\bar{a} - \bar{b}\right)\left(\left(\bar{a}.\bar{a}\right)\bar{b} - \left(\bar{a}.\bar{b}\right)\bar{a} + 2(\bar{a}.\bar{b})\bar{b} - 2(\bar{b}.\bar{b})\bar{a}\right)$

$= \left(2\bar{a} - \bar{b}\right)\left(\bar{b} - 0 + 0 - 2\bar{a}\right)$

$= -4\bar{a}.\bar{a} - \bar{b}.\bar{b} = -5$.

77. If $\dfrac{dy}{dx} = y + 3 > 0$ and y(0) = 2, then y(ln 2) is equal to

(1) 7 (2) 5

(3) 13 (4) -2

Key: (1)

Sol.: $\dfrac{dy}{dx} = y + 3$

$\dfrac{dy}{y+3} = dx$

On integrating

ln |y + 3| = x + c

$\Rightarrow$ ln (y + 3) = x + c

Since y(0) = 2

$\Rightarrow$ c = ln 5

ln (y + 3) = x + ln 5

put x = ln 2

y = 7.

78. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3, 1) and has eccentricity $\sqrt{2/5}$ is

(1) $3x^2 + 5y^2 - 32 = 0$

(2) $5x^2 + 3y^2 - 48 = 0$
(3) $3x^2 + 5y^2 - 15 = 0$
(4) $5x^2 + 3y^2 - 32 = 0$

Key: (1)

Sol.: Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

It passes through (-3, 1) so $\frac{9}{a^2}+\frac{1}{b^2}=1$ ... (i)

Also, $b^2 = a^2 (1 - 2/5)$

$\Rightarrow 5b^2 = 3a^2$ ... (ii)

Solving we get $a^2 = \frac{32}{3}$, $b^2 = \frac{32}{5}$

So, the ellipse is $3x^2 + 5y^2 = 32$.

79. If the mean deviation about the median of the numbers a, 2a, ... , 50a is 50, then |a| equals

(1) 2 (2) 3
(3) 4 (4) 5

Key: (3)

Sol.: Median is the mean of 25th and 26th observation

$\therefore M = \frac{25a+26a}{2} = 25.5\,a$

$M.D\,(M) = \frac{\Sigma|x_i - M|}{N}$

$\Rightarrow 50 = \frac{1}{50}\,[2\times|a|\times(0.5 + 1.5 + 2.5 + ...\ 24.5)]$

$\Rightarrow 2500 = 2|a| \times \frac{25}{2}(25)$

$\Rightarrow |a| = 4$.

80. $\lim\limits_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) does not exist (2) equals $\sqrt{2}$
(3) equals $-\sqrt{2}$ (4) equals $\frac{1}{\sqrt{2}}$

Key: (1)

Sol.: Let x - 2 = t

$\lim\limits_{t\to 0}\frac{\sqrt{1-\cos 2t}}{t}$

$= \lim\limits_{t\to 0}\sqrt{2}\frac{|\sin t|}{t}$

Clearly R.H.L. = $\sqrt{2}$

L.H.L. = $-\sqrt{2}$

Since R.H.L. ≠ L.H.L. So, limit does not exist.

81. Statement-1:
The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$

Statement-2:
The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true.

Key: (1)

Sol.: The number of ways of distributing n identical objects among r persons such that each person gets at least one object is same as the number of ways of selecting (r - 1) places out of (n-1) different places, that is ${}^{n-1}C_{r-1}$.

82. Let R be the set of real numbers.
Statement-1:
A = {(x, y) ∈ R × R : y - x is an integer} is an equivalence relation on R.
B = {(x, y) ∈ R × R : x = αy for some rational number α} is an equivalence relation on R.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true.

Key: (3)

Sol.: Clearly, A is an equivalence relation but B is not symmetric. So, it is not equivalence.

83. Consider 5 independent Bernoulli's trails each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval.

(1) $\left(\frac{1}{2},\frac{3}{4}\right]$ (2) $\left(\frac{3}{4},\frac{11}{12}\right]$
(3) $\left[0,\frac{1}{2}\right]$ (4) $\left(\frac{11}{12},1\right]$

Key: (3)

Sol.: P(at least one failure)
= 1 - P (No failure)
= $1 - p^5$

Now $1 - p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \left(\frac{1}{2}\right)^5$

$\Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

So, P lies in the interval $[0, \frac{1}{2}]$.

84. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ ($c > 0$) touch each other if
(1) $2|a| = c$ (2) $|a| = c$
(3) $a = 2c$ (4) $|a| = 2c$

Key: (2)

Sol.: If the two circles touch each other, then they must touch each other internally.

So, $\frac{|a|}{2} = c - \frac{|a|}{2}$

$\Rightarrow |a| = c$.

85. Let A and B be two symmetric matrices of order 3.
Statement-1:
A(BA) and (AB)A are symmetric matrices.
Statement-2:
AB is symmetric matrix if matrix multiplication of A and B is commutative.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key: (2)

Sol.: Given, $A' = A$
$B' = B$
Now $(A(BA))' = (BA)'A' = (A'B')A' = (AB)A = A(BA)$
Similarly $((AB)A)' = (AB)A$
So, A(BA) and (AB)A are symmetric matrices.
Again $(AB)' = B'A' = BA$
Now if $BA = AB$, then AB is symmetric matrix.

86. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is
(1) $P(C|D) = P(C)$ (2) $P(C|D) \geq P(C)$
(3) $P(C|D) < P(C)$ (4) $P(C|D) = \frac{P(D)}{P(C)}$

Key: (2)

Sol.: $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \geq P(C)$

(Since $0 < P(D) \leq 1$

So, $\frac{P(C)}{P(D)} \geq P(C)$)

87. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to
(1) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (2) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$
(3) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (4) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Key: (4)

Sol.: $\vec{a}.\vec{b} \neq 0$
$\vec{a}.\vec{d} = 0$
$\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$
$\Rightarrow \vec{b} \times (\vec{c} - \vec{d}) = 0$
$\vec{b}$ is parallel to $\vec{c} - \vec{d}$
$\vec{c} - \vec{d} = \lambda\vec{b}$
Taking dot product with $\vec{a}$
$\vec{a}.\vec{c} - 0 = \lambda\vec{a}.\vec{b}$
$\Rightarrow \lambda = \frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}$
So, $\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

88. Statement-1:
The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
Statement-2:
The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key: (2)

Sol.: The direction ratio of the line segment joining points A(1, 0, 7) and B(1, 6, 3) is 0, 6, -4.
The direction ratio of the given line is 1, 2, 3.
Clearly $1 \times 0 + 2 \times 6 + 3 \times (-4) = 0$
So, the given line is perpendicular to line AB.
Also , the mid point of A and B is (1, 3, 5) which lies on the given line.
So, the image of B in the given line is A, because the given line is the perpendicular bisector of line segment joining points A and B.

89. If $A = \sin^2 x + \cos^4 x$, then for all real x:

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$

(3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

Key: (1)

Sol.: $A = \sin^2 x + \cos^4 x$

$= \sin^2 x + \cos^2 x. (1 - \sin^2 x)$

$= 1 - \frac{1}{4} \sin^2 2x$

Since; $0 \le \sin^2 2x \le 1$

So, $\frac{3}{4} \le A \le 1$.

90. The number of values of k for which the linear equations

$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$

posses a non-zero solution is:

(1) 3 (2) 2
(3) 1 (4) zero

Key: (2)

Sol.: For the system to have non-zero solution

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow k^2 - 6k + 8 = 0$

$\Rightarrow k = 2$ or 4.

Read the following instructions carefully:

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet**(Side–1)** with **Blue / Black Ball Point Pen**.
2. For writing / marking particulars on **Side–2** of the Answer Sheet, use **Blue / Black Ball Point Pen only**.
3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet / Answer Sheet.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each **incorrect response, one-fourth (1/4)** of the total marks allotted to the question would be deducted from the total score. **No deduction** from the total score, however, will be made **if no response** is indicated for an item in the Answer Sheet.
6. Handle the Test Booklet and Answer Sheet with care, as under no circumstance *(except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.*
7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations / writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Apace for Rough Work'. This space is given at the bottom of each page and in 3 pages (Page 21 – 23) at the end of the booklet.
8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. **However, the candidates are allowed to take away this Test Booklet with them**.
9. Each candidate must show on demand his/her Admit Card to the Invigilator.
10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.
11. The candidates should be leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet**.
12. Use of Electronic / Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.
13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall / room.**

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