## I B.Tech Regular Examinations,June 2010 MATHEMATICS-1

Common to ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE,CE

Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions <br> All Questions carry equal marks

1. (a) Test the convergence of the series $\sqrt{n+1}-\sqrt{n-1}$
(b) Test the convergence of the series $u_{n}=\frac{(n+1)^{n} \cdot x^{n}}{n^{n+1}}$
(c) For what values of the following series is convergent $-x+\frac{x^{2}}{2^{2}}-\frac{x^{2}}{2^{2}}+\frac{x^{3}}{3^{2}}-\frac{x^{4}}{4^{2}}+\ldots$
2. (a) Find the radius of curvature at the origin for the curve $x^{4}-y^{4}+x^{3}-y^{3}+$ $x^{2}-y^{2}+y=0$
(b) Find the centre of curvature on $x=a(\theta+\sin \theta), y=a(1-\cos \theta) \quad[8+7]$
3. (a) Find a unit normal vector to the surface $x^{3}+y^{3}+z^{3}=3$ at the point $(1,-2,1)$
(b) Applying, Green's theorem evaluate $\int(y-\sin x) d x+\cos x d y$, where C is the plane triangle enclosed by the lines $y=0, x=\frac{\pi}{2}$ and $y=\frac{2 x}{\pi} \quad[8+7]$
4. (a) Using mean value theorem, For $0<a<b$ Prove that $1-\frac{a}{b}<\log \frac{b}{a}<\frac{b}{a}-1$ and hence show that $\frac{1}{6}<\log \frac{6}{5}<\frac{1}{5}$
(b) Expand $e^{x} \sin y$ in powers of x and y .
5. (a) Find the length of the cycloid $\mathrm{x}=\mathrm{a}(\theta+\sin \theta) \mathrm{y}=\mathrm{a}(1-\cos \theta)$ between two consecutive cusps. Show that the length of the arc of the cycloid between the points $\theta=0$ and $\theta=2 \Psi$ is given by $\mathrm{s}=4$ asin $\psi$. Show further that for this curve $s=\sqrt{8 a y}$.
(b) Evaluate the double integral $\int_{0}^{a} \int_{0}^{\sqrt{a^{2}-y^{2}}}\left(x^{2}+y^{2}\right) d y d x$
6. (a) Find $L[g(t)]$ where $\begin{aligned} g(t)= & \cos \left(t-\frac{2}{3} \pi\right) t>2 \pi / 3 \\ & =0, t<2 \pi / 3\end{aligned}$
(b) Find $L^{-1}\left[\frac{s}{\left(s^{2}+1\right)\left(s^{2}+9\right)\left(s^{2}+25\right)}\right]$
7. (a) Form the differential equation by eliminating arbitrary constants $y=C e^{\sin ^{-1} x}$
(b) Solve the differential equation $\left(x^{2}+2 y^{2}\right) d x-x y d y=0$
(c) If the air is maintained at $25^{\circ} \mathrm{C}$ and the temperature of the body cools from $100^{\circ}$ to $80^{\circ} \mathrm{C}$ in 10 minutes, find the temperature of the body after 20 minutes and when the temperature will be $40^{\circ} \mathrm{C}$.
8. (a) Solve the differential equation $\left(D^{2}+9\right) y=\cos 3 x+\sin 2 x$
(b) A mass of 4 lbs suspended from a light elastic string of natural length 3 feet extends it to a distance 2 ft . One end of the string is fixed and a mass of 2 lbs is attached to other. The mass is held so that the string is just un stretched and is then let go. Find the amplitude, period and the maximum velocity of the S.H.M.
