Code No: R05010102 **Set No. 1**

I B.Tech Supplimentary Examinations, Aug/Sep 2007
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the following series $\sum\left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$ [5]
   (b) Find the interval of convergence of the series whose nth term is $\sum \frac{(-1)^n(x+2)}{(2^n+5)}$ [5]
   (c) If a < b prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. Deduce the following [6]
      i. $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\Pi}{4} + \frac{1}{6}$
      ii. $\frac{5\pi+4}{20} < \tan^{-1}2 < \frac{\pi+2}{4}$
2. (a) If x = r sinθ cosϕ, y = r sinθ sinϕ and z = r cosθ prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$. [6]
   (b) Find the radius of curvature at any point on the curve y = c cosh$\frac{x}{c}$. [10]
3. Trace the curve y = a cosh (x/a) and find the volume got by rotating this curve about the x-axis between the ordinates x = ± a. [16]
4. (a) Form the differential equation by eliminating the arbitrary constant secy + secx = c + x^2/2.
   (b) Solve the differential equation: (2y sin x + cos y) dx = (x siny + 2 cosx + tany) dy.
   (c) Find the orthogonal trajectories of the family: r^n sin nθ = b^n. [3+7+6]
5. (a) Solve the differential equation: y$''$ - 4y$'$ + 3y = 4e^{3x}, y(0) = - 1, y$'$(0) = 3.
   (b) Solve the differential equation: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\,cos\,\log(1+x)$ [8+8]
6. (a) Find the Laplace Transformation of the following function: t e^{-t} sin2t.

(b) Using Laplace transform, solve $y'' + 2y' + 5y = e^{-t} \sin t$, given that $y(0) = 0$, $y'(0) = 1$.

(c) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\, dxdy$ [5+6+5]

7. (a) Prove that div($\mathbf{A}\times\mathbf{B}$)=$\mathbf{B}$.curl$\mathbf{A}$ - $\mathbf{A}$.curl$\mathbf{B}$.

   (b) Find the directional derivative of the scalar point function ϕ (x,y,z) = $4xy^2 + 2x^2yz$ at the point
   A(1, 2, 3) in the direction of the line AB where B = (5,0,4). [8+8]

8. Verify Stoke's theorem for the vector field $\mathbf{F}=(2x-y)\mathbf{i}-yz^2\mathbf{j}-y^2z\mathbf{k}$ over the upper half surface of $x^2+y^2+z^2=1$, bounded by the projection of the xy-plane. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$. [5]
   (b) Find the interval of convergence of the series $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty$. [5]
   (c) Write Taylor's series for f(x) = $(1-x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval [0,1]. [6]

2. (a) Locate the stationary points and examine their nature of the following functions:
   u = $x^4 + y^4 - 2x^2 + 4xy - 2y^2$, (x > 0, y > 0).
   (b) From any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, perpendiculars are drawn to the coordinates axes. Prove that the envelope of the straight line joining the feet of these perpendiculars is the curve. $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ [8+8]

3. (a) Trace the curve $9ay^2 = x\,(x-3a)^2$.
   (b) Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]

4. (a) Form the differential equation by eliminating the constant $x^2+y^2-2ay=a^2$.
   (b) Solve the differential equation $\frac{dy}{dx}\,(x^2 + y^3 + xy) = 1$.
   (c) If the air is maintained at 15 ^oC and the temperature of the body cools from 70 ^{o}C to 40 ^{o}C in 10 minutes, find the temperature after 30 minutes.[3+7+6]

5. (a) Solve the differential equation: $(D^3 - 1)y = e^x + \sin^3 x + 2$.
   (b) Solve the differential equation: $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$. [8+8]

6. (a) Solve the differential equation $\frac{d^2x}{dx^2} + 9x = sin\,t$ using Laplace transforms given that x(0) = 1, $x'(0) = 0$
   (b) Change the order of integration hence evaluate $\int_0^1 \int_{x^2}^{2-x} x\,dydx$ [8+8]

7. (a) If $\phi_1 = x^2\, y$ and $\phi_2 = xz - y^2$ find $\nabla\times\,(\,\nabla\phi_1 \times \nabla\phi_2\,)$

   (b) If $\overline{F} = (3x^2 + 6y)\, i - 14yz\, j + 20\, x\, z^2\, k$ evaluate the line integral $\int\limits_C \overline{F}\,.\,d\,\overline{r}$ from (0,0,0), (1,1,1) along x = t, y = t, z = t^3. [8+8]

8. Verify divergence theorem for F = 6zi + (2x + y)j − xk, taken over the region bounded by the surface of the cylinder $x^2 + y^2 = 9$ included in z = 0, z = 8, x = 0 and y = 0. [16]

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Code No: R05010102 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test for convergence of the series $\sum_{\infty}^{1}\left[\sqrt{n^4+1}\ -\ \sqrt{n^4-1}\right]$ [5]

   (b) Find the interval of convergence of the following series
   $\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+$ .... [5]

   (c) Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}\ >\ \cos^{-1}\frac{3}{5}\ >\ \frac{\pi}{3}-\frac{1}{8}$ using Lagrange's mean value theorem. [6]

2. (a) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid of revolution $4x^2 + 4y^2 + 9z^2 = 36$. [8+8]

   (b) Find the envelope of the family of curves $\frac{ax}{\cos\alpha}-\frac{by}{\sin\alpha}=a^2-b^2$, α is a parameter.

3. (a) Trace the curve r=a sin2θ.

   (b) Find the whole length of the curve $8a^2y^2$=$x^2(a^2$–$x^2)$. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constants, y=a secx+ b tan x.

   (b) Solve the differential equation (y^4+2y)dx+(xy^3+2y^4-4x)dy=0.

   (c) If 30% of a radio active substance disappears in 10 days, how long will it take for 90% to disappear. [3+7+6]

5. (a) Solve the differential equation: $(D^2 + 4D + 4)y = 18$ coshx.

   (b) Solve the differential equation: $(D^2 + 4)y =$ cosx. [8+8]

6. (a) Show that L{t^n f(t)}= $(-1)^n\,\frac{d^n}{ds^n}\ \left[\ \overline{f}(s)\right]$ where n = 1,2,3, ...

   (b) Evaluate: $L^{-1}\left[\frac{1}{s^2(s+2)}\right]$

   (c) Evaluate $\int\int$r sinθ dr dθ over the cardioid r = a(1 - cosθ) above the initial line. [5+6+5]

7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2 + y^2 + z^2}$
   (b) Find constants a, b, c so that the vector $\mathbf{A}$ =(x+2y+az)$\mathbf{i}$ +(bx–3y–z)$\mathbf{j}$+(4x+cy+2z)$\mathbf{k}$ is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$. [8+8]

8. Verify Green's theorem for $\oint_C [(3x - 8y^2)dx + (4y - 6xy)dy]$
   where C is the region bounded by x=0, y=0 and x + y = 1. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the following series
$1 + \frac{3}{1}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + ... \; x > 0$ [5]
(b) Test the following series for absolute /conditional convergence
$\sum \frac{(-1)^n.n}{3n^2-2}$ [5]
(c) Expand e^x secx as a power series in x up to the term containing x^3 [6]

2. (a) Find the points on the surface z^2=xy+1 that are nearest to the origin.
(b) Prove that if the centre of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at one end of the minor axis lies at the other end, then the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$. [8+8]

3. (a) Trace the lemniscate of Bernoulli : $r^2 = a^2 \cos 2\theta$.
(b) The segment of the parabola $y^2 = 4ax$ which is cut off by the latus rectum revolves about the directrix. Find the volume of rotation of the annular region. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : y^2=4ax.
(b) Solve the differential equation: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)\, e^x \sec y$.
(c) In a chemical reaction a given substance is being converted into another at a rate proportional to the amount of substance unconverted. If $(1/5)^{th}$ of the original amount has been transformed in 4 minutes how much time will be required to transform one half. [3+7+6]

5. (a) Solve the differential equation $y'' - y' - 2y = 3e^{2x}, y(0) = 0, y'(0) = 2$
(b) Solve the differential equation: $(D^2+1)y =$ cosec x by variation of parameters method. [8+8]

6. (a) Find the Laplace Transformations of the following functions
$e^{-3t}(2\cos 5t - 3\sin 5t)$

(b) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

(c) Evaluate: $\int\limits_0^1 \int\limits_0^{\sqrt{1+x^2}} \frac{dx\,dy}{(1+x^2+y^2)}$ [5+6+5]

7. (a) Find a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4ax^2y + z^3 = 4$ cut orthogonally at (1, -1, 2).

   (b) Show that $\overline{F} = (2xy + z^3)\,i + x^2\,j + 3\,x\,z^2\,k$ is a conservative force field. Find the scalar potential and the work done by F in moving an object in this field from (1, –2, 1) to (3,1,4). [8+8]

8. Verify divergence theorem for F = 4xz i – y^2 j + yz k, where S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. [16]

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