Code No: 07A3BS02 **Set No. 1**

II B.Tech I Semester Regular Examinations, November 2008
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Electronics & Computer Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\int_0^1 y^{q-1} (\log \frac{1}{y})^{p-1} dy = \frac{\Gamma(p)}{q^p}$ where p>0, q>0.
   (b) Prove that $\beta(m, \frac{1}{2}) = 2^{2m-1} \beta(m, m)$ [8+8]
2. (a) Prove that the function f(z) = u + i v , where $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, $z \neq 0$ and f(0) = 0 is continuous and that Cauchy's Riemann equations are satisfied at the origin, yet f′(z) does not exist.
   (b) Find the analytic function whose real part is y +e^x cos y. [8+8]
3. (a) Prove that
      i. $i^i = e^{-(4n+1)\frac{\pi}{2}}$
      ii. $\log i^i = -(2n + \frac{1}{2})\pi$
   (b) If tan(A + iB) = x + iy , prove that x^2 + y^2 + 2x cot2A = 1. [8+8]
4. (a) Evaluate $\int_0^{3+i} z^2 dz$, along
      i. the line y = x/3
      ii. the parabola x =3y^2
   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{z^3-2z+1}{(z-i)^2} dz$, where c is the circle $|z| = 2$. [10+6]
5. (a) Expand $f(z) = \frac{1+2z}{z^2+z^3}$ in a series of positive and negative powers of z.
   (b) Expand e^z as Taylor's series about z = 1. [8+8]
6. Evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ using residue theorem. [16]
7. Use Rouche's theorem to show that the equation z^5 + 15z + 1 = 0 has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$ [16]
8. Show that the transformation $w = z + \frac{1}{z}$, converts that the radial lines θ= constant in the z-plane in to a family of confocal hyperbolar in the w-plane. [16]

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Set No. 2

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate
   i. $\int_0^\infty \sqrt{x}\, .\, e^{-x^3}\, dx$
   ii. $\int_0^\infty t^{-3/2}\,(1 - e^{-t})\, dt$
   iii. $\int_0^1 x^4\,[\log(\frac{1}{x})]^3\; dx$

   (b) Show that $\int_b^a (x - b)^{m-1}(a - x)^{n-1}\, dx = (a - b)^{m+n-1}\,\beta(m, n)$ [12+4]

2. (a) Prove that the function f(z) = $\sqrt{|xy|}$ is not analytic at the origin even though the C - R equations are satisfied thereat.

   (b) Find the analytic function whose real part is y / (x^2 + y^2). [8+8]

3. (a) Separate into real and imaginary parts of cosh (x + iy) .

   (b) Find all the roots of the equation
   i. sin z = cosh 4
   ii. sin z = i. [8+8]

4. (a) Prove that
   i. $\int_c \frac{dz}{z-a} = 2\pi i$
   ii. $\int_c (z - a)^n dz = 0$, [n, any integer ≠ -1]

   (b) State and prove Cauchy's integral theorem. [8+8]

5. (a) Give two Laurent's series expansions in powers of z for $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which these expansions are valued.

   (b) Expand $f(z) = \frac{1}{z^2-3z+2}$ in the region
   i. $0 < |z - 1| < 1$
   ii. $0 < |z| < 2$ [8+8]

6. (a) State and prove Cauchy's Residue theorem.

(b) Find the residue at z = 0 of the function
$f(z) = \frac{1+e^z}{\sin z + z\cos z}$ [8+8]

7. State Rouche's theorem. Prove that $z^7 - 5z^3 + 12 = 0$ all the roots of this equation lie between the circles $|z| = 1$ *and* $|z| = 2$ [16]

8. (a) Find and plot the image of the regions
   i. $x > 1$
   ii. $y > 0$
   iii. $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$

   (b) Prove that every bilinear transformation maps the totality of circle and straight lines in the z - plane on to the totality of circles and straight lines in the w-plane. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate $\int_0^\infty x^2 . e^{-x^8} dx \times \int_0^\infty x^2 . e^{-x^4} dx$
   (b) Show that $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$
   (c) Prove that $\int_0^1 \frac{x}{\sqrt{(1-x^5)}} dx = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$ [6+5+5]

2. (a) Show that $f(z) = \frac{xy^2 (x+iy)}{x^2+y^4}$, $z \neq 0$ and f(0) = 0 is not analytic at z=0 although C- R equations are satisfied at the origin.
   (b) If w = φ+iψ represents the complex potential for an electric field and ψ= 3x^{3}y − y^3 find φ. [8+8]

3. (a) Find the real part of the principal value of $i^{\log(1+i)}$
   (b) Separate into real and imaginary parts of sech (x + i y) . [8+8]

4. (a) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path
      i. y = x
      ii. y = x^2
   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ where c is the circle $|z| = 1$ [8+8]

5. (a) Expand log (1 - z) when $|z| < 1$
   (b) Determine the poles of the function
      i. $\frac{z}{cosz}$
      ii. cot z. [8+8]

6. Show by the method of residues, $\int_0^\pi \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). [16]

7. (a) Apply Rouche's theorem to determine the number of roots (zeros) of P(z) = z^4 - 5z + 1, with in annulus region $1 < |z| < 2$.

(b) Evaluate $\oint\limits_{C} \frac{f^1(z)}{f(z)} dz$ where c is a simple closed curve, where $f(z) = \frac{z^2-1}{(z^2+z)^2}$, where c: $|z| = 2$ [16]

8. (a) Show that horizantal lines in z - plane are mapped to ellipser in w - plane under the transformation w = sin z.

   (b) Define Bilineer transformation. Determine the Bilinear transformation which maps z = 0, -i, 2i into w = 5i, ∞, $\frac{-i}{3}$ [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Prove that $\Gamma(m)\,\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$
   (b) Express the following integrals in terms of gamma function:
      i. $\int\limits_0^\infty \frac{x^c}{c^x}\,dx$
      ii. $\int\limits_0^\infty a^{-bx^2}\,dx$ [6+10]

2. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v(x, y) to be analytic in the region R, are
   (a) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R.
   (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ [16]

3. (a) Separate into real and imaginary parts of coth z
   (b) If tan log (x + i y) = a + i b where $a^2 + b^2 \neq 1$, show that tan log ($x^2 + y^2$) = $\frac{2a}{i-a^2-b^2}$ [8+8]

4. (a) Evaluate $\int\limits_0^{1+i} (x^2+iy)\,dz$ along the path y = x and y = x^2.
   (b) Evaluate, using Cauchy's integral formula $\int\limits_c \frac{e^{2z}}{(z-1)(z-2)}dz$, where c is the circle $|z| = 3$ [8+8]

5. (a) Expand $f(z) = \frac{z-1}{z+1}$ in Taylor's series about the point z = 0 and z = 1.
   (b) Determine the poles of the function $f(z) = \frac{1-e^{2z}}{z^4}$ [8+8]

6. (a) Determine the poles of the function $f(z) = \frac{z^2}{(z+1)^2(z+2)}$ and the residues at each pole.
   (b) Evaluate $\oint\limits_c \frac{dx}{(z^2+4)^2}$ where $c = |z-i| = 2$ [8+8]

7. Show that the polynomial $z^5 + z^3 + 2z + 3$ has just one zero in the fist quadrant of the complex plane. [16]

8. (a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$

   (b) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscate $p^2 = \cos 2\,\phi$. [8+8]

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